IV *A Propoſition on the* Balance, *not taken Notice of by Mechanical Writers, explain'd and confirm'd by an Experiment before the Royal Society, by* J. T. Desaguliers, L. L. D. *F. R. S.*

THO' the following Theorem is agreeable to, and deducible from, Mechanical Principles, yet as it has not been taken Notice of by Mechanical Writers, though often talk'd of among Handicraft Workmen, I thought it might not be improper to take Notice of it here, and to make an Experiment agreeable to the Demonſtration.

Theorem, *Figure* 1.

A B is a Balance, on which is ſuppos'd to hang at one End B the Scale E with a Man in it, who is counterpoiſed by the Weight W hanging at A, the other End of the Balance. I ſay, that if ſuch a Man, with a Cane or any rigid ſtreight Body, puſhes upwards againſt the Beam any where between the Points C and B (provided he does not puſh directly againſt B) he will thereby make himſelf heavier, or over-poiſe the Weight W, though the Stop G G hinders the Scale E from being thruſt outwards fromwards C towards G G. I ſay likewiſe, That if the Scale and Man ſhould hang from D, the Man by puſhing upwards againſt B, or any where between B and D (provided he does not puſh directly againſt D) will make himſelf lighter, or be be over-poiſed by the Weight W, which did before only counterpoiſe the Weight of his Body and the Scale.

If the common Center of Gravity of the Scale E, and the Man ſuppoſed to ſtand in it be at *k*, and the Man by thruſting againſt any Part of the Beam, cauſe the Scale to move outwards ſo as to carry the ſaid common Center of Gravity to *k x*, then inſtead of B E, L *l* will become the Line of Direction of the compound Weight, whoſe Action will be encreaſed in the Ratio of L C to B C. This is what has been explain'd by ſeveral Writers of Mechanicks; but no one, that I know of, has conſider'd the Caſe when the Scale is kept from flying out, as here by the Poſt G G, which keeps it in its Place, as if the Strings of the Scale were become inflexible. Now to explain this Caſe, let us ſuppoſe the Length B D of half of the Brachium B C to be equal to 3 Feet, the Line B E to 4 Feet, the Line E D of 5 Feet to be the Direction in which the Man puſhes, D F and F E to be reſpectively equal and parallel to B E and B D, and the whole or abſolute Force with which the Man puſhes, equal to (or able to raiſe) 10 Stone. Let the oblique Force E D (= 10 Stone) be reſolv'd into the two E F and E B, (or its Equal F D) whoſe Directions are at right Angles to each other, and whoſe reſpective Quantities (or Intenſities) are as 6 and 8, becauſe E F and B E are in that Proportion to each other, and to E D. Now ſince E F is parallel to B D C A, the Beam, it does no way affect the Beam to move it upwards; and therefore there is only the Force repreſented by F D, or 8 Stone to puſh the Beam upwards at D. For the ſame Reaſon, and becauſe Action and Reaction are equal, the Scale will be puſh'd down at E with the Force of 8 Stone alſo. Now ſince the Force at E pulls the Beam perpendicularly downwards from the Point B, diſtant from C the whole Length of the Brachium BD, its Action downwards will not be diminiſhed, but may be expreſs'd by $\overline{8 \times BC}$: Whereas the Action upwards againſt D will be half loſt, by reaſon of the diminiſh'd Diſtance from the Center, and is only to be expreſs'd by $\overline{8 \times \frac{BC}{2}}$; and when the Action upwards to raiſe the Beam is ſubſtracted from the Action downwards to depreſs it, there will ſtill remain 4 Stone to puſh down the Scale; becauſe $\overline{8 \times BC} - \overline{8 \times \frac{BC}{2}} = 4\,BC$. Conſequently a Weight of 4 Stone muſt be added at the End A to reſtore the Æquilibrium. *Therefore a Man, &c. puſhing upwards under the Beam between* B *and* D, *becomes heavier.* Q. E. D. On the contrary, if the Scale ſhould hang at F from the Point D, only 3 Feet from the Center of Motion C, and a Poſt *g g* hinders the Scale from being puſh'd inwards towards C, then if a Man in this Scale F puſhes obliquely againſt B with the oblique Force above-mention'd; the whole Force, for the Reaſons before given (in reſolving the oblique Force into two others acting in Lines perpendicular to each other) will be reduc'd to 8 Stone, which puſhes the Beam directly upwards at B, while the ſame Force of 8 Stone draws it directly down at D towards F. But as C D is only equal to half of C B, the Force at D compar'd with that at B, loſes half its Action, and therefore can only take off the Force of 4 Stone from the Puſh upwards at B; and conſequently the Weight W at A will preponderate, unleſs an additional Weight of 4 Stone be hanged at B. *Therefore a Man*, &c. *puſhing upwards under the Beam between* B *and* D *becomes lighter*. Which was alſo to be demonſtrated.

SCHOLIUM I.

Hence knowing the abſolute Force of the Man that puſhes upwards, (that is, the whole oblique Force) the Place of the Point of Truſion D, and the Angle made by the Direction of the Force with a Perpendicular to the Beam at the ſame Point, we may have a general Rule to know what Force is added to the End of the Beam B in any Inclination of the Direction of the Force or Place of the Point D.

RULE *for the firſt Caſe.*

Firſt find the perpendicular Force by the following Analogy, whoſe Demonſtration is known to all that underſtand the Application of oblique Forces.

As the Radius:
To the right Sine of the Angle of Inclination::
So is the oblique Force:
To the perpendicular Force.

Then the perpendicular Force multiplied into the Length of the Brachium B C, *minus* the ſaid Force multiplied into the Diſtance D C, will give the Value of the additional Force at B, or of the Weight requir'd to reſtore the Æquilbirium at A.

Or to expreſs it in the Algebraical Way. Let *o f* expreſs the oblique Force, *p f* the Perpendicular Force,

and

and x the Force requir'd, or Value of the additional Weight at A to reſtore the Æquilibrium.

$$DE : DF :: of : pf$$

$$\overline{pf \times BC} - \overline{pf \times DC} = x$$

The ſame Rule will ſerve for the ſecond Caſe, if the Quantity found be made negative, and the additional Weight ſuſpended at B. Or having found the Value of the Perpendicular Force, the Æquation will ſtand thus $-\overline{pf \times BC} + \overline{pf \times DC} = -x$, and conſequently the additional Weight muſt be hanged at B; becauſe $-x$ at A is the ſame as $+x$ at B.

SCHOLIUM II.

Hence it follows alſo, that if, in the firſt Caſe, the Point of Truſion be taken at C, the Force at B, (or Force whoſe Value is requir'd) will be the whole Perpendicular Force; becauſe C D is equal to nothing: And if the Point D be taken beyond C towards A; the Perpendicular Force puſhing upwards at that *Point*, multiplied into D C, muſt be added to the ſame Force multiplied into B C, that is $\overline{pf \times BC} + \overline{pf \times DC} = x$.

The Machine I made uſe of to prove this experimentally, was as follows. Fig. 2. The Braſs Balance A B is 12 Inches long, moveable upon the Center C, with a Perpendicular Piece *B b* hanging at the End B, and moveable about a Pin at B, and ſtopp'd at its lower End *b* (by the upright Plate G G) from being thruſt out of the Perpendicular by the puſhing Pipe F E, whoſe lower Point being put into a little Hole at H, the upper Wire or Point (when put into another little Hole under the Beam at D) is by Means of the

Worm-

Worm-ſpring E F preſſing againſt the Plug E to drive forwards the ſaid Wire *b* D, made to puſh the ſaid Beam upwards with the Force of the Spring. T S S is a Stand, to which is fix'd the Pillar T C that ſuſtains the Balance; and it has alſo a Slit SS to receive a Shank of the moveable Plate G G, to be fix'd in any Part of the Slit by a Screw underneath.

EPERIMENT.

Hang on B *b*, as in the Figure. Then let E F be ſo applied to the Hole H, that its upper Wire *b* D *k* may go through a little Loop at D ſo as not to thruſt the Beam upwards, but be in the ſame Poſition as if it did, that by hanging on the Weight W, the Brachium B C with B *b* and F E may be counterpois'd; and then the Action againſt D and H may be eſtimated without the Weight of the puſhing Pipe.

Then drawing down the End of the Wire *k*, thruſt it into the little Hole under D, and B will be ſo pull'd downwards as to require the additional Weight of 4 Ounces to be hung on at A to reſtore the Æquilibrium, when B H is 4 Inches, B D 3 Inches, and the whole Force of the Spring equal to 10 Ounces.

I need not here ſay, that for explaining the ſecond Caſe, B *b* is to be ſuſpended at D, with the Plate G G fix'd to ſtop it at the Place M to keep it from being puſh'd towards T, and that the upper End of G F E D *k* muſt puſh into an Hole made under B, in which Caſe the Weight P muſt be hang'd at B to reſtore the Æquilibrium.

P. S.

P. S. To ſhew experimentally that the Force which the Spring exerts in this oblique Truſion is equal to 10 Ounces: Take the Beam A B, which weighs 4 Ounces, from its Pedeſtal C T, and having ſuſpended at each End, A and B 3 Ounces, ſupport it under its Center of Gravity by the puſhing Pipe E F ſet upright under it, and you will find that the Beam with the two Weights will thruſt in the Wire *k h* as far as *h*, the Place which the oblique Truſion drives it to.

Philo: Trans: N^{o} 409.

Fig. 1.

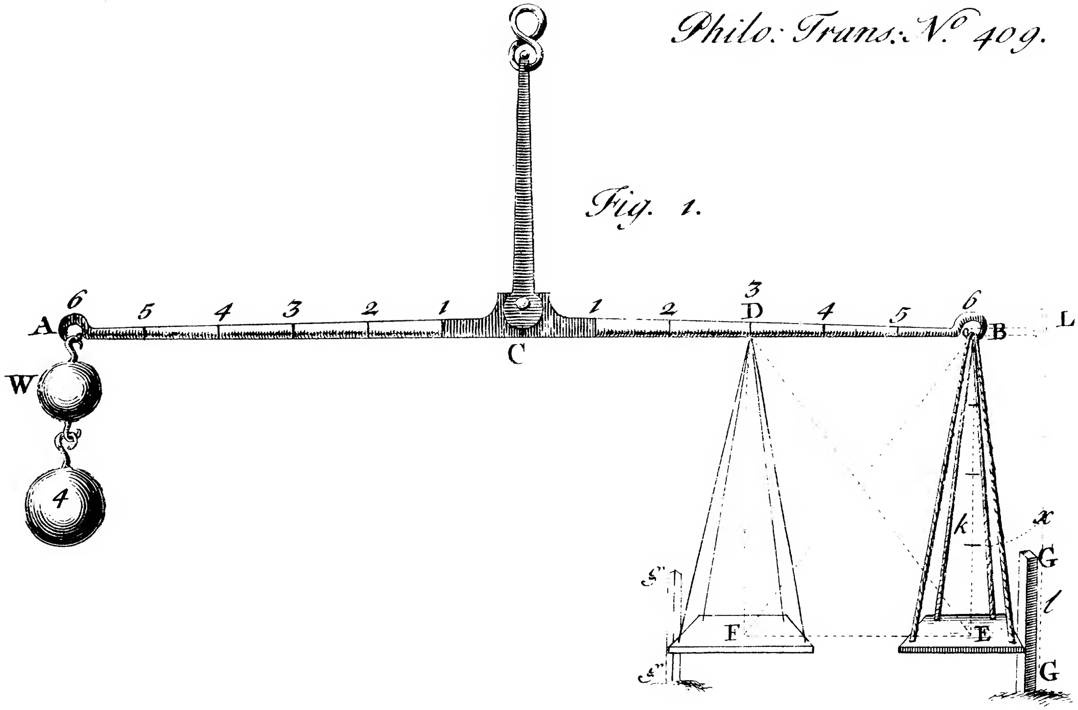

Fig. 2.